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NEW METHOD FOR ANALYSING THE SPATIAL STRUCTURE OF EUROPE

***Abstract**: Many theoretical and practical works aim at describing the spatial structure of Europe. As spatial relations undergo continuous change, their analysis is always justifiable. In our study, we give an overview of the models describing the spatial structure of Europe and highlight their diversity by discussing some of them, without any claim to completeness. Our study aims at describing the economic spatial structure of Europe with a bi-dimensional regression analysis based on the gravitational model. With the help of the gravity shift-based model, we can clearly justify the appropriateness of models based on different methodological backgrounds.*

***Key words**: spatial models, gravity model, bi-dimensional regression, Europe.*

JEL Classification: R10

Introduction

Some of the theories and models describing the social and economic spatial structure of Europe are static. They deal with the current status and with structures. Among others, the 'European Backbone' of Brunet (1989) – later called the 'Blue Banana' – and the 'Central European Boomerang' of Gorzelak (2012) fall into this category (Fig. 1). Moreover, the models that aim at visualising different polygons (triangles, tetragons) also belong to this category. See Brunet (2002).

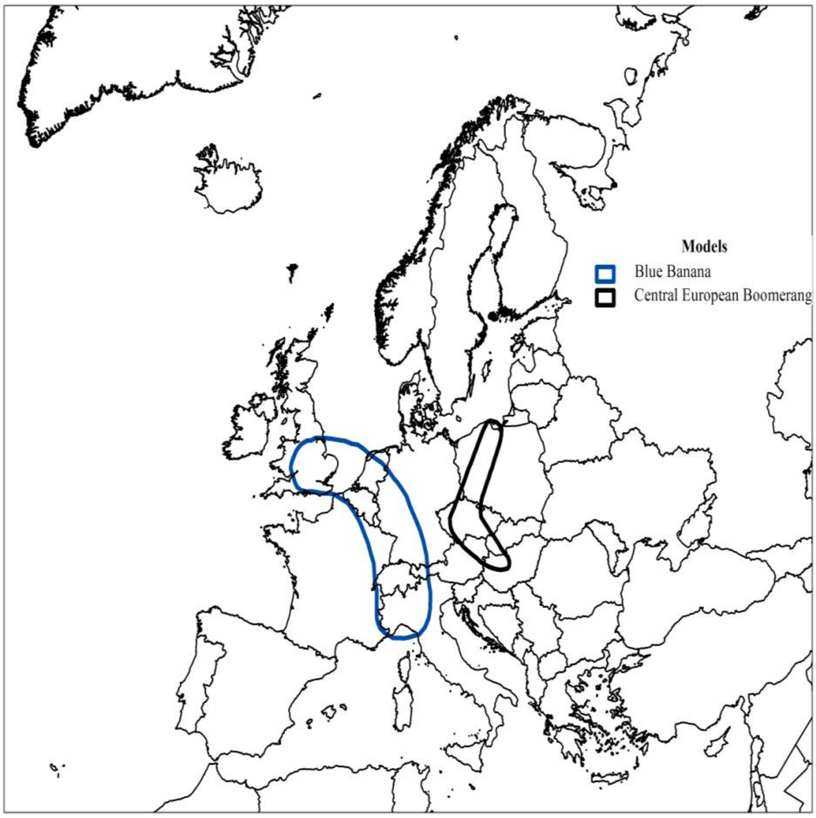

Figure 1: Spatial structure models (1)
Source: own compilation based on Brunet (1989) and Gorzelak (2012)

Another type of popular spatial structure models is formed by visualisations that emphasise potential movements and changes in spatial structure and development form. Some of them are presented here, without any claim to completeness. One of them is the growing zone at the northern shore of the Mediterranean Sea called the European Sunbelt by Kunzmann (1992) (in Kozma 2003).

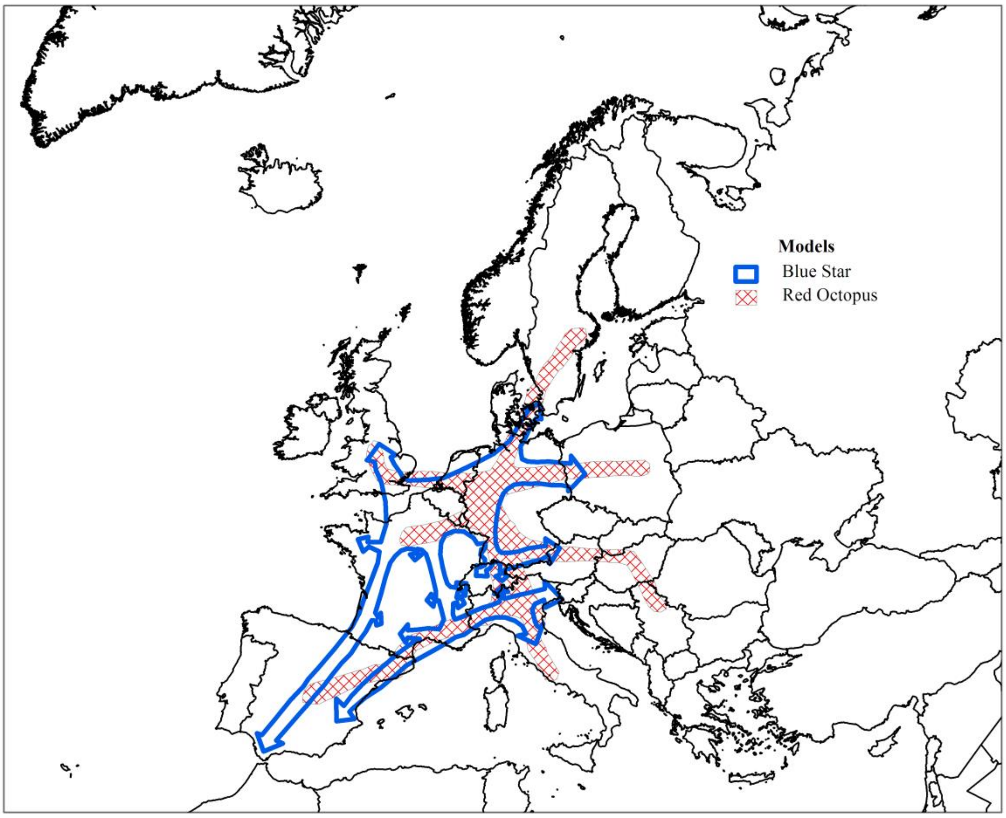

Figure 2: Spatial structure models (2)
Source: own compilation based on van der Meer (1998) and Dommergues (1992)

The model of the so-called 'Red Octopus' can definitely be classified as a dynamic model provided that it focuses on the future and introduces potential changes in the future, given that this is a vision for 2046 about the regions of Europe that will develop most rapidly (van der Meer 1998) (Fig. 2). This form includes the group of developed zones and their core cities. Development can also be visualised by the so-called 'Blue Star' (Dommergues 1992), which includes arrows to demonstrate the directions of development and the dynamic areas (Fig. 2).

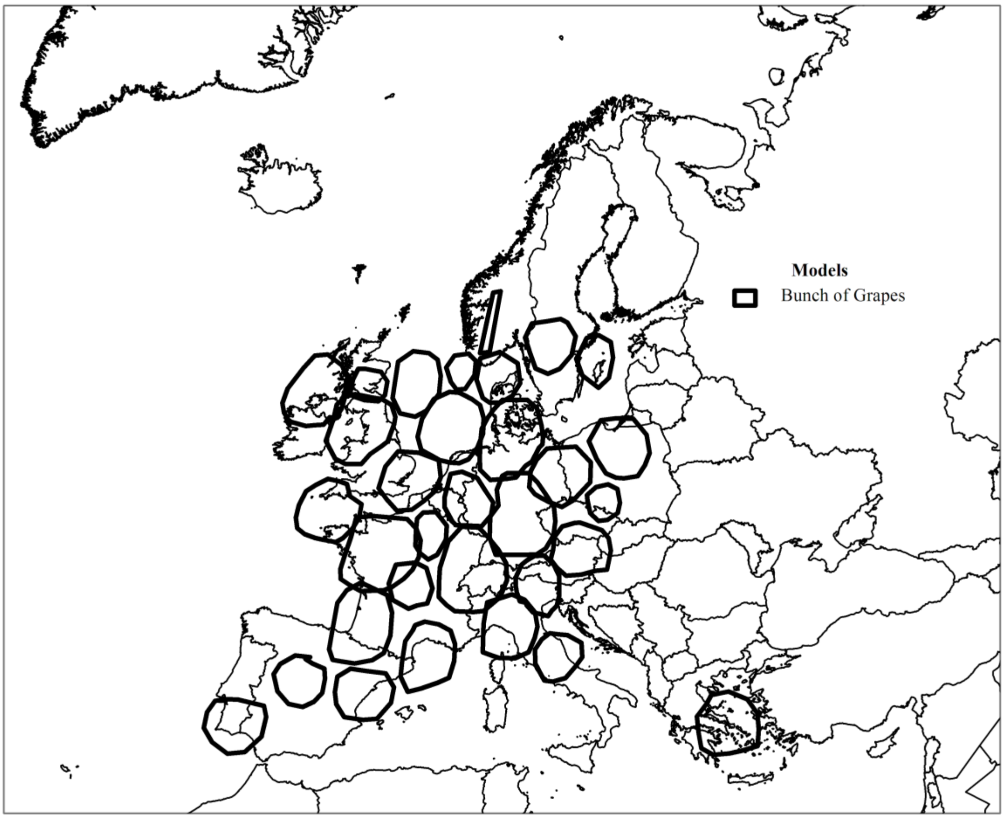

Figure 3: Spatial structure models (3)
Source: own compilation based on Kunzmann (1992); Kunzmann and Wegener (1991, 1996).

We argue that the so-called 'Bunch of Grapes' model of Kunzmann (1992) and Kunzmann and Wegener (1991, 1996) includes change as well as the visualisation of development (Fig. 3). Focusing on the polycentric spatial structure means that urban development and the dynamic change of urban areas can be emphasised (Szabó 2009). Therefore we argue that polycentricity has rightly become an increasingly popular idea and a key part of the European Spatial Development Perspective (ESDP 1999). It plays an increasingly important role in European cohesion policy as well (Faludi 2005; Kilper 2009). At the same time, however, critical statements appear against this kind of approach to planning, like from the point of view of economic efficiency or sustainable development (Vandermotten et al. 2008).

In many cases, the crucial question is not the form describing the spatial structure or the quality and the extension of the formation, i.e. the static description. Rather, the crucial questions are about the visualisation of the changes, processes and the relationships among regions. In addition, it is important to analyse the methods and developments that can provide an opportunity to utilise advantages and positive effects. Dynamic visualisations can contribute to this process.

In the following sections we examine more thoroughly the background of the spatial structural relations and models described above with the application of the gravity model and bidimensional regression.

1. Gravity models and examination of the spatial structure

Besides potential models, the other approach to examining spatial structure is gravity models, which are based on the application of physical forces. With the approach that we present here, one can assign attraction directions to the given territorial unit that are caused by other units. This method complements and specifies the view of spatial structure described by the potential models.

The universal gravitational law, Newton's gravitational law, states that any two point-like bodies mutually attract each other by a force whose magnitude is directly proportional to the product of the bodies' weight and is inversely proportional to the square of the distance between them (Budo 1970) (Eq. 1):

$$F = \gamma \cdot \frac{m_1 \cdot m_2}{r^2} \tag{1}$$

where the proportionality measure γ is the gravitational constant (regardless of space and time).

If the radius vector from point mass 2 to point mass 1 is designated by r, then the unit vector from point 1 to point 2 is —***r*** and therefore the gravitational force applied on point mass 1 owed to point mass 2 is (MacDougal 2013):

$$\vec{F}_{1,2} = -\gamma \cdot \frac{m_1 \cdot m_2}{r^2} \cdot \frac{\vec{r}}{r} \tag{2}$$

A gravitational force field is definite if the direction and the size of gradient K can be defined at each point of the given field. To do so, provided that K is a vector, three pieces of data are necessary in each point (two in the case of a plane), such as the rectangular components K_x, K_y, K_z of the gradient as the function of the place. Many force fields, however, like the gravitational force field, can be described in a much simpler way, that is, instead of three variables, just one scalar function, the so-called potential (Fig. 4) (Budó 1970).

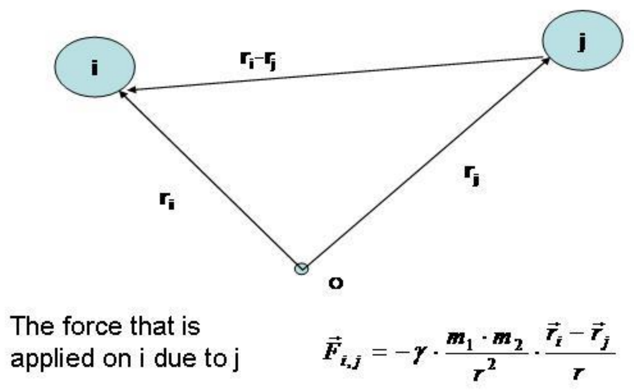

Figure 4: Calculation of the gravitational force

Potential has a similar relation to gradient as the work or potential energy has to force. If in the gravitation field of gradient K the trial mass on which a force of ***F***=*m**K*** is applied is moved from point A to point B by force ***-F*** (without acceleration) along with some curve, then

$$L = -\int_A^B F_s ds$$

has to be employed against force ***F*** based on the definition of work. This work is independent of the curve from A to B. Therefore it is the change of the potential energy of an arbitrary trial mass:

$$L = E_{potB} - E_{potA} = -\int_A^B F_s ds = -m\int_A^B K_s ds .$$

By dividing by *m*, we obtain the potential difference between points B and A in the gravitational space:

$$U_B - U_A = -\int_A^B K_s ds$$

In most social scientific applications of the gravitational model the space was primarily intended to be described by only one scalar function (see for example the potential model) (Kincses & Tóth 2011), whereas in the gravitational law it is mainly the vectors characterising the space that have an important role. The main reason for this is that arithmetical operations with numbers are easier to handle than calculations with vectors. In other words, for work with potentials, solving the problem also means avoiding calculation problems.

Albeit potential models often show properly the concentration focus of the population or GDP and the space structure, they are not able to provide any information on the direction towards which the social attributes of the other regions attract a specified region or on the force with which they attract it.

Therefore, by using vectors we are trying to demonstrate in which direction the European regions (NUTS 2) are attracted by other regions in the gravity space compared with their real geographical position. With this analysis it is possible to reveal the centres and fault lines representing the most important areas of attractiveness and it is possible to visualise the differences among the gravitational orientations of the regions, which we will describe in more detail in a later section. First of all, let us look at the method.

In the traditional gravitational model (Stewart 1948) the 'population force' between i and j is expressed in D_{ij}, where W_i and W_j are the populations of the settlements (regions), d_{ij} is the distance between i, and j and g is the empirical constant:

$$D_{ij} = g \cdot \left(\frac{W_i \cdot W_j}{d_{ij}^2}\right) \qquad (3)$$

With the generalisation of the above formula, the following relationship is obtained in Eqs. (4) and (5):

$$D_{ij} = \left| \vec{D}_{ij} \right| = \frac{W_i \cdot W_j}{d_{ij}^{c}} \tag{4}$$

$$\vec{D}_{ij} = -\frac{W_i \cdot W_j}{d_{ij}^{c+1}} \cdot \vec{d}_{ij} \tag{5}$$

where W_i and W_j indicate the masses taken into consideration, d_{ij} is the distance between them and c is the constant, which is the change in the intensity of the inter-territorial relations as a function of the distance. With the increase of power, the intensity of the inter-territorial relations becomes more sensitive to the distance and, at the same time, the importance of the masses gradually decreases (see Dusek 2003).

With this extension of the formula not only the force between the two regions but also its direction can be defined. In the calculations it is worth dividing the vectors into x and y components and then summarising them separately. In order to calculate this effect (the horizontal and vertical components of the forces), the necessary formulas can be deduced from Eqs. (4) and (5):

$$D_{ij}^{X} = -\frac{W_i \cdot W_j}{d_{ij}^{c+1}} \cdot (x_i - x_j) \tag{6}$$

$$D_{ij}^{Y} = -\frac{W_i \cdot W_j}{d_{ij}^{c+1}} \cdot (y_i - y_j) \tag{7}$$

where x_i, x_j, y_i, y_j are the coordinates of centroids of regions i and j.

If, however, the calculation is carried out for each region included in the analysis, the direction and the force of the effect on the given territorial unit can be defined with Eqs. (8) and (9).

$$D_{ij}^{X} = -\sum_{j=1}^{n} \frac{W_i \cdot W_j}{d_{ij}^{c+1}} \cdot (x_i - x_j) \tag{8}$$

$$D_{ij}^{Y} = -\sum_{j=1}^{n} \frac{W_i \cdot W_j}{d_{ij}^{c+1}} \cdot (y_i - y_j) \tag{9}$$

With these equations, the magnitude and the direction of the force owed to the other regions can be defined in each territorial unit. The direction of the vector assigned to the regions determines the attraction direction of the other regions, and the magnitude of the vector is related to the magnitude of the force. In order to make visualisation possible the forces are transformed to proportionate movements in Eqs. (10) and (11):

$$x_i^{mod} = x_i + \left(D_{ij}^{X} * \frac{x^{max}}{x^{min}} * k \frac{1}{\frac{D_{ij}^{X\,max}}{D_{ij}^{X\,min}}} \right) \tag{10}$$

$$y_i^{mod} = y_i + \left(D_{ij}^{Y} * \frac{y^{max}}{y^{min}} * k \frac{1}{\frac{D_{ij}^{Y\,max}}{D_{ij}^{Y\,min}}} \right) \tag{11}$$

where X_i mod and Y_i mod are the coordinates of the new points modified by gravitational force, x and y are the coordinates of the original point set, their extreme values are x_{max}, y_{max}, x_{min}, and y_{min}, D_{ij} are the forces along the axes and k

is a constant whose value in this case is 0.5. We obtained this value as a result of an iteration process.
It is worth comparing the point set obtained by the gravitational calculation (using population, number of employees, and GDP as a weight) with the baseline point set, that is, with the actual real-world geographic coordinates (and later with each other) and examining how the space is changed and distorted by the field of force. In this comparison, not only may the conventional gravitational fields be located as shown in other models, but we can find the gravity direction too. With this analysis it is possible to reveal the centres and fault lines representing the most important areas of attractiveness and to visualise the differences among the gravitational orientations of the regions. In order to realise this in practice, bidimensional regression needs to be used.

2. Bidimensional regression

It is possible to compare the new point set with the original one by applying this analysis. This comparison can naturally be done with visualisation, but in the case of such a large number of points, it probably does not provide a really promising result by itself. Much more favourable results can be obtained by applying bidimensional regression analysis (see the equations related to the Euclidean version in Table 1), which is a quantifiable method. In this examination, we apply population, number of employees, and GDP as weighting variables.

Table 1. Equations of bidimensional Euclidean regression

1. Regression equation	$\begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{pmatrix} \beta_1 & -\beta_2 \\ \beta_2 & \beta_1 \end{pmatrix} * \begin{pmatrix} X \\ Y \end{pmatrix}$
2. Scale difference	$\Phi = \sqrt{\beta_1^2 + \beta_2^2}$
3. Rotation	$\Theta = \tan^{-1}\left(\frac{\beta_2}{\beta_1}\right)$
4. β_1	$\beta_1 = \frac{\sum(a_i - \bar{a})*(x_i - \bar{x}) + \sum(b_i - \bar{b})*(y_i - \bar{y})}{\sum(x_i - \bar{x})^2 + \sum(y_i - \bar{y})^2}$
5. β_2	$\beta_2 = \frac{\sum(b_i - \bar{b})*(x_i - \bar{x}) - \sum(a_i - \bar{a})*(y_i - \bar{y})}{\sum(x_i - \bar{x})^2 + \sum(y - \bar{y})^2}$
6. Horizontal shift	$\alpha_1 = \bar{a} - \beta_1 * \bar{x} + \beta_2 * \bar{y}$

7. Vertical shift	$\alpha_2 = \bar{b} - \beta_2 * \bar{x} - \beta_1 * \bar{y}$
8. Correlation based on error terms	$r = \sqrt{1 - \frac{\sum\left[(a_i - a_i)^2 + (b_i - b_i')^2\right]}{\sum\left[(a_i - \bar{a})^2 + (b_i - \bar{b})^2\right]}}$
9. Resolution difference of a square sum	$\sum\left[(a_i - \bar{a})^2 + (b_i - \bar{b})^2\right] = \sum\left[(a_i' - \bar{a})^2 + (b_i' - \bar{b})^2\right] + \sum\left[(a_i - a_i')^2 + (b_i - b_i')^2\right]$ SST=SSR+SSE
10. A ‘	$A' = \alpha_1 + \beta_1(X) - \beta_2(Y)$
11. B ‘	$B' = \alpha_2 + \beta_2(X) + \beta_1(Y)$

Sources: Tobler (1994) and Friedman and Kohler (2003) cited by Dusek (2011, p. 14).

In the equations in Table 1, x and y refer to the coordinates of the independent form, a and b designate the coordinates of the dependent form, a' and b' are the coordinates of the independent form in the dependent form. α_1 refers to the extent of the horizontal shift, and α_2 defines the extent of the vertical shift. β_1 and β_2 are used to determine the scale difference (Φ) and Θ is the rotation angle. SST is the total sum of squares, SSR is the sum of squares owed to regression and SSE is the explained sum of squares of errors/residuals that is not explained by the regression. To visualise the bidimensional regression the Darcy program[1] is useful. The grid is fitted to the coordinate system of the dependent form and its interpolated modified position makes it possible to further generalise the information about the points of the regression.

3. Empirical analysis

Our analysis can be carried out at the NUTS2 level. Comparison of the results (between real and modified coordinates) with those of bidimensional regression can be found in Table 2.

[1] http://spatial-modelling.info/Darcy-2-module-de-comparaison

Table 2. Bidimensional regression between gravitational and geographical spaces

Indicator	r	α1	α2	β1	β2	Φ	Θ	SST	SSR	SSE
Population	0.97	0.02	0.08	1.00	0.00	1.00	-0.34	53 272	52 643	629
Employment	0.97	0.02	0.06	1.00	0.00	1.00	-0.75	51 959	51 446	493
GDP	0.97	0.06	0.04	1.00	0.00	1.00	0.62	51 974	51 480	494

There is no significant difference in the gravitational shifts created by using the different variables, which is indicated by the equally high values of the two-dimensional correlation (r). Its highest value can be one, which is reached when the exact coordinates of the points coincide with each other as a result of motion, rotation and rescaling. The minimal value of correlation is zero, which means that each point of a point pattern has the same coordinate. In our case, the difference between the geographical and gravitational coordinates is minimal. α_1 refers to the extent of the horizontal shift, whereas α_2 defines the extent of the vertical shift. The horizontal shift is the highest in the case of GDP, whereas the vertical shift is the highest in the case of the calculation using the population. β_1 and β_2 are used to determine the scale difference (Φ) and the rotation angle (Θ). In our analysis, a difference could only be found in the rotation angle. If Θ = 0, the XY coordinate system does not need to be rotated. If it is equal to zero, this means a clockwise rotation. It is also the case in our analysis that rotation is a little higher for GDP than for the two other variables. Theoretically, decomposition of the total sum of squares is carried out in the same way as for a univariate case and the notations are the same (SST: total sum of squares, SSR: sum of squares owed to regression, SSE: explained sum of squares of errors/residuals).

For the map application of the bidimensional regression, the Darcy program can be usefully applied (http://www.spatial-modelling.info/Darcy-2-module-decomparaison)

The arrows in Figs. 5 to 7 show the direction of movement and the grid colour refers to the nature of the distortion. Warm colours indicate divergence; that is, movements in the opposite direction, which can be considered to indicate the most important gravitational fault lines. Areas indicated in green and its shades refer to the opposite, namely to concentration, to the movements in the same directions (convergence), which can be considered to be the most important gravitational centres.

The visualised analysis of the bidimensional analysis with three variables has slightly different results. Analysis using the population clearly highlights the most important actors of the European demographic space structure and the most populated, decisively urban areas. As regards the number of employees, the spatial picture is quite similar. Deviations in this aspect are slightly smaller and the extent of concentration is slightly more modest. As a result of the calculation using GDP the number of nodes decreases significantly. In the map with contour lines, the regions related to the so-called Blue Banana space structure – the economic engine of the European Union –emerge unambiguously. Within this area, two centres can be identified. On the one hand, the regions of southern England, the Benelux states and northern France make up the most important node, whereas in the case of the regions of northern Italy and southern Germany (and the related regions of Switzerland), a central position exists, but to a lesser extent. This area emerges also as a result of the calculations carried out with the two other variables. In those cases, other areas are linked to it.

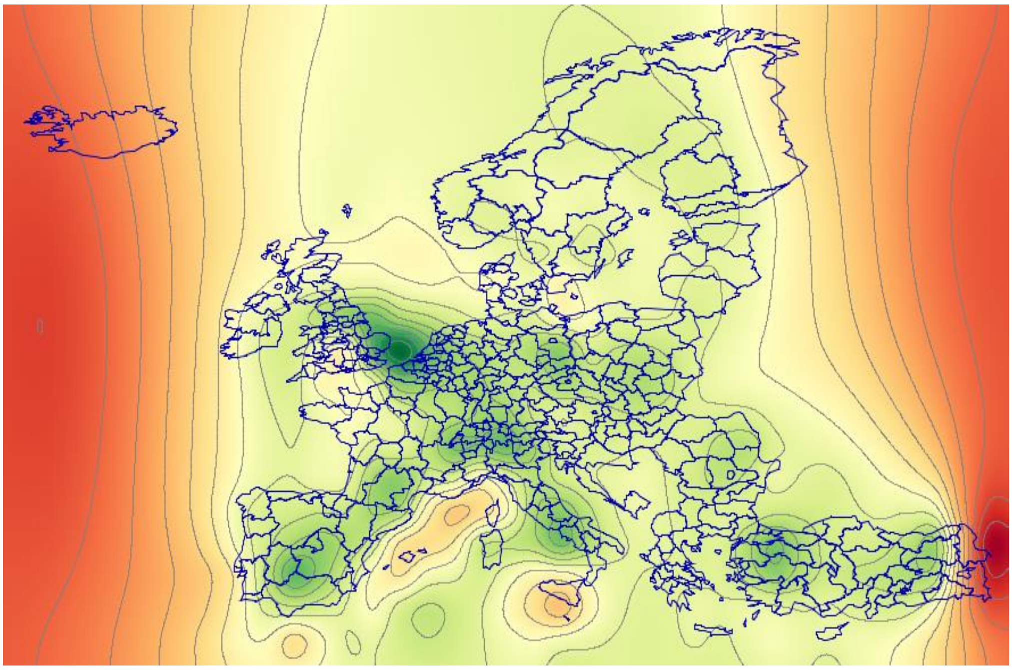

Figure 5. Directions of skewness of the gravitational space compared with the geographic space in the case of European (NUTS2) regions (mass factor: population)

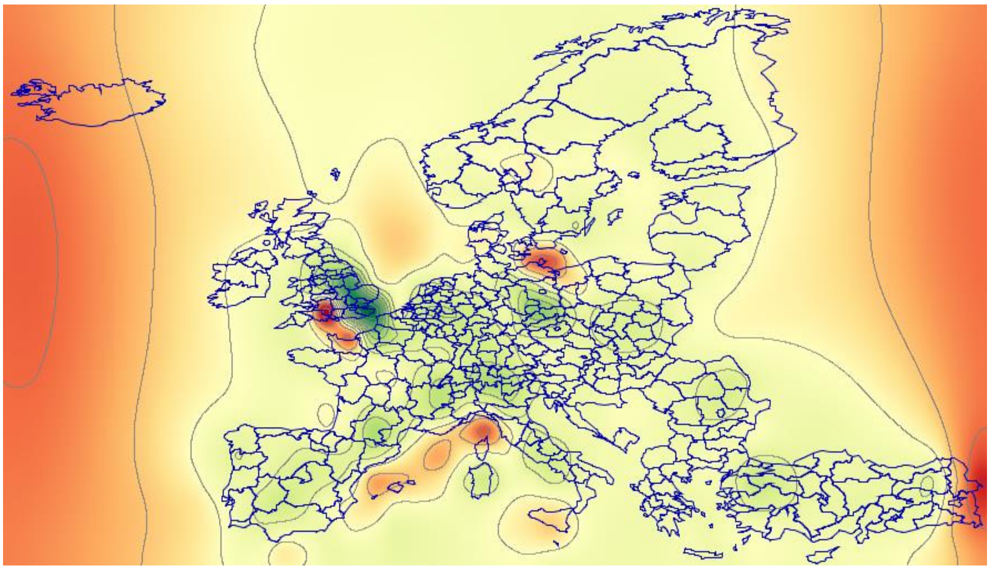

Figure 6. Directions of skewness of the gravitational space compared with the geographic space in the case of European (NUTS2) regions (mass factor: number of employees)

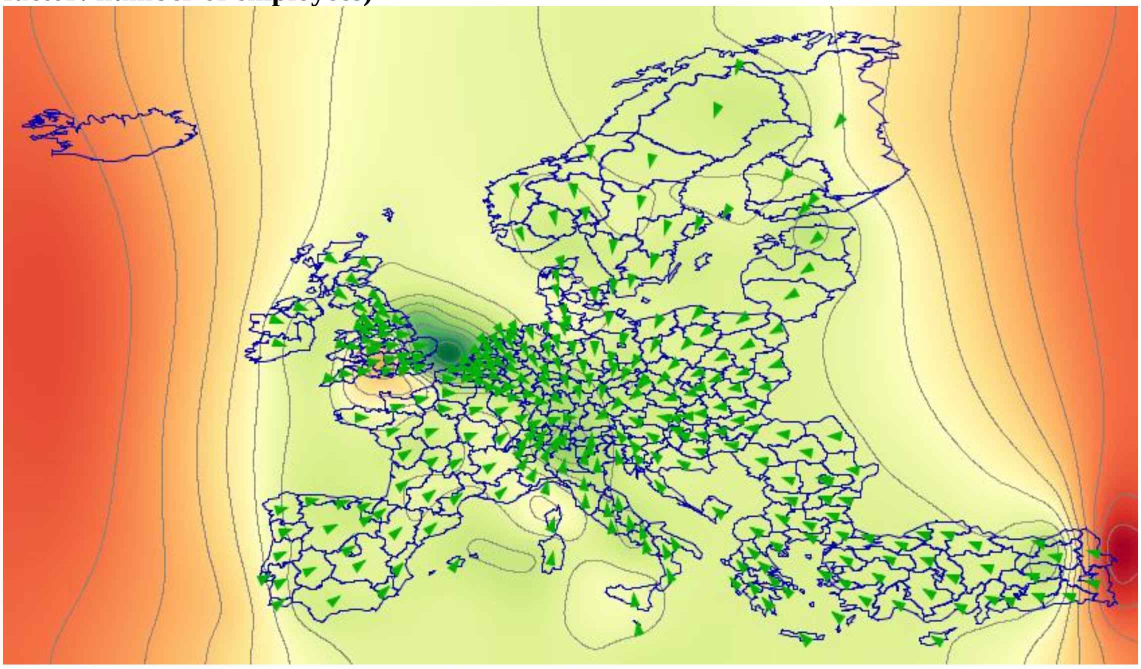

Figure 7. Directions of skewness of the gravitational space compared with the geographic space in the case of European (NUTS2) regions (mass factor: GDP)

Defining the core regions is easy with gravity analysis, provided that these are defined as regions with converging spatial movements that can be considered the main gravitational centres. These regions are shown in green.

In the following section our investigation has mainly concentrated on the economic structure of Europe. Therefore we tried to take into account the change in the economic structure, using only the GDP data. To do so, the gravity calculations were performed for 1995 and 2009. In this calculation, we cannot include the regions of Turkey, so the figures of 2009 are slightly different from the ones described, such as those in Fig. 8. In order to measure changes, we compare and analyse the two sets of gravity points (1995 and 2009). The two-dimensional regression calculations are shown below (Tables 3 and 4).

Table 3. Bidimensional regression between gravitational and geographical spaces

Year	r	α1	α2	β1	β2	Φ	Θ	SST	SSR	SSE
1995	0.92	0.07	0.37	0.99	0.00	0.99	0.00	65 446	62 525	2 922
2009	0.92	0.05	0.26	0.99	0.00	0.99	0.00	65 632	62 811	2 821

Table 4. Bidimensional regression between gravitational spaces

Year	r	α1	α2	β1	β2	Φ	Θ	SST	SSR	SSE
1995/2009	0.99	- 0.01	- 0.06	1.00	0.00	1.00	0.00	65 632	65 607	25

Our results show that there is a strong relationship between the two point systems; the transformed version from the original point pile can be obtained without using rotation (Θ = 0). No essential ratio difference between the two shapes is observed.

In terms of change from 1995 to 2009, 15 gravity centres are shown on the map in red ellipses (Fig. 8). They show a crucial part of the economic potential of large cities. Such hubs are in the surroundings of Rome, Marseille, Madrid, Vienna, Hamburg, Brussels, Olso, Glasgow, etc. A gravity 'breakline' can be seen in northern France, northern Italy, Switzerland, Hessen in Germany and Northern Saxony.
In general, the change was not widespread in the examined period but rather focused on only a few areas. These areas are parts of the bunch of grapes fields, which may show the increasing importance of this theory. We cannot see, however, as many nodes or 'grapes' as the model predicts.

As far as the analysis of change is considered, we can find the closest connection to the Red Octopus model, because 11 out of the 15 gravity nodes were directly affected by the octopus arms. We can confirm the favourable position of the regions concerned and the unfavourable position in one region with other models – the Sunbelt zone, the French Banana, the German hump and the Pentagon theories – but we cannot justify the existence of the Eastern European Boomerang.

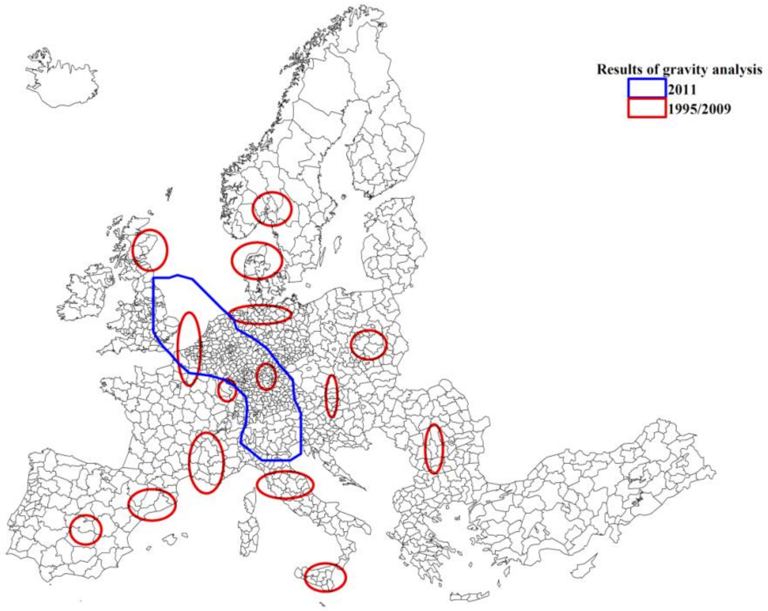

Figure 8: The results of the gravity method

4. Summary

In our research we first introduced the most important models for investigating the spatial structure of Europe, which were then compared with the results of our gravity calculations. From the latest population, number of employees and GDP calculations we analysed the spatial structure of Europe. The results definitely verify the banana shape. The European core area, based on our analysis, still has the banana shape, as other authors have concluded, but the different analyses highlight the existence of related regions that are moving to catch up.

We draw our conclusions on the basis of static and dynamic gravity calculations. Our model justifies mostly the Red Octopus theory in terms of the change in GDP. Our findings clearly outline the banana shape in the European spatial structure that has long been dominant.

Recent developments have been able to alter these fundamental spatial relations very slightly; no radical modification can be observed. We believe that the European spatial structure is likely to remain unchanged in the medium term, although we may see more changes in position than between 1995 and 2009. For this reason, we believe that similar analysis will definitely be necessary in the future.

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